Samuelson and the Non-Substitution Theorem: Some Methodological Remarks

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With his revealed preference theory (RPT), Samuelson intended to offer "operational" foundations to neoclassical consumer theory by getting rid of unobservable or ill-conceived psychological arguments like, especially, introspection. According to Samuelson, RPT is operational inasmuch as utility functions are now based only on observable elements, namely consumed bundles of goods. In this article, we show that the same methodological process was implicitly applied by Samuelson to neoclassical production theory, and especially the production function. After defining and discussing Samuelson's operationism, we offer a methodological interpretation of Samuelson's "non-substitution theorem" (NST) as an operational theorem. We aim at showing that the same methodological process rules RPT and NST: while in RPT observable elements are bundles of goods consumed, in NST these elements are bundles of inputs consumed so as to measure technical coefficients; from observable choices by consumers and producers, one derives the corresponding behavioral or technological function, respectively the utility function and the production function. Therefore, both functions are operational concepts that offer operational foundations to both standard microeconomic analysis of consumption through indifference curves (deduced from the utility function), and to analysis of production through isoquants (deduced from the production function). Although Samuelson's application of his operational-methodology-to-consumer theory has been studied at length, its application to production theory has been neglected.

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"The world takes people too much at their words." (Samuelson, 1962, p.10)

When dealing with the Pareto-Walrasian general equilibrium theory, Pascal Bridel repeatedly underlined that from one generation of economists to another, the methodology ruling this theory has been continuously changing: from Léon Walras's idealism to Vilfredo Pareto's positivism; from Paul A. Samuelson's operationism[1] to Robert E. Lucas's instrumentalism; and so forth (Bridel, 1999, 2007). In this regard, one major methodological turn was the move from cardinalist utility functions, resting on psychological foundations, to ordinalist ones. Samuelson's contribution, both methodological (introducing operationism in economics) and theoretical (the well-known revealed-preference theory), is probably the most important

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[1] In philosophy of science, operationism is synonymous to operationalism. See Bridgman (1927, 1938) and Carnap (1936, 1937).

(Bridel, 1999), although recent works in economics increasingly reintroduce psychological elements in economic theory (Bridel and Salvat, 2004). In our article we attempt at drawing a parallel between Samuelson's operationalization-of-consumer theory through revealed preference theory, and his works in production theory from the late 1940s on, in particular the non-substitution theorem.

In 1949, the non-substitution theorem (NST) was first demonstrated simultaneously and independently by Samuelson and Georgescu-Roegen for a so-called (static)[2] "generalized model of Leontief." The theorem raised the question as to whether substitutions among factors of production could not occur even if possible. Samuelson and Georgescu-Roegen showed that, although possible, in certain conditions if the structure of final demand changes, the structure of production remains unchanged and no factor substitution occurs. These special conditions are, in particular, that the economy admits *(i)* at most one primary factor, i.e., one non-produced factor of production like land; and *(ii)* no joint production, i.e., each technology produces only one outcome. The dual interpretation of the non-substitution theorem asserts that "under certain specified conditions an economy will have one particular price structure for each admissible value of the profit rate, regardless of the pattern of the final demand" (Salvadori, 1987, p.680).

As a matter of fact, such specific conditions are verified in the "*simple* models of Leontief," the famous input-output Leontief models (1937, 1944, 1949). Therefore, in such a model, it is relevant to assume, like in Walras's "Theory of production" in the *Éléments d'économie politique pure* (1874–1877), constancy of technical coefficients and thus to rule out substitutions phenomenon. However, in a "*generalized* model of Leontief" (Arrow 1951, Dantzig 1949; Dantzig and Wood 1949; Koopmans 1951a; Samuelson 1951), one assumes multiple primary factors and joint products, and thus factor substitution occurs and, as a consequence, marginal productivity theory is verified. To Samuelson, classical political economy falls under the specific (restrictive) assumptions of the NST, while Pareto-Walrasian general-equilibrium theory falls under the generalized system. Furthermore, according to Samuelson, the generalized model is the relevant one to describe real economies:

> Ricardo and Smith would probably have admitted that the relative prices of joint products—of venison and skin, for example—would have to be determined by a demand theory and not from labor and land costs alone. One wonders why they did not worry about this "jointness," which every student of Walrasian equilibrium knows to be an intrinsic part of the actual pricing relations among diverse factors and goods. (Samuelson, 1959, p.18)

[2] All the analyses developed here belong to the static and a-temporal framework.

In a nutshell, from a theoretical point of view, non-substitution then "is here to remind us that it is not easy to escape the imperialism of the general equilibrium theory" (d'Autume, 1990, p.246). The non-substitution theorem has eventually become one of the cornerstones of post-Walrasian general-equilibrium theory of production (Arrow, 1983; McKenzie, 2002; Weintraub, 1983).

Our article deals only with Samuelson's NST. We do not discuss the theoretical relevance of the NST for political economy. Instead, we are interested in the possible relationship between, on the one hand, Samuelson's well-known operationist methodology (see Blaug, 1980; Cohen, 1995) that he developed about consumer theory, in particular the revealed preference theory (Samuelson, 1938a, 1938b, 1938c, 1998), and, on the other hand, in the realm of production theory, his demonstration of the non-substitution theorem. We ask the following question: given Samuelson's operationist epistemology, is the NST an operational theorem? We argue in favor of a positive answer.

1. "A Theorem Concerning Substitutability in Open Leontief Models"

The non-substitution theorem was first demonstrated[3] for a so-called "generalized open model of Leontief" (Samuelson, 1951), which is an *n*-industries full-employment general-equilibrium model with exogenous final demand.[4] In order to "generalize" Leontief's model,[5] Samuelson took the same set of assumptions as Leontief except for the constancy of technical coefficients: the generalized-production function allows substitutions among factors of production. This is a *generalized model of Leontief*.

Let X_i be the total output of industry *i*, for all $i = 1 \ldots n$; x_{ij} output of the *i-th* industry used by industry *j*; a_{ij} is the quantity of input *i* used to produce one unity of output *j*; and C_i the final demand ("final output") addressed to the *i-th* industry. For each industry, equilibrium quantity is distributed such as: $X_i = \sum_{j=1}^{n} x_{ij} + C_i$. Labor is the $n+1^{\text{th}}$ commodity and is the only

[3] Samuelson's demonstration was first presented in 1949 at the "Linear Programming" conference organized at Chicago by the Cowles Commission under the direction of Tjalling C. Koopmans. Samuelson's proof was published in 1951 in the Cowles Commission monograph *Activity Analysis*, edited by Koopmans, which gathered a selection of the linear programming conference papers. Afterwards, Samuelson published a series of papers to develop the theoretical consequences of his theorem (Samuelson, 1957, 1959a, 1959b). He also expanded the NST to an intertemporal framework (Samuelson, 1961).

[4] The non-substitution theorem was first named "substitution theorem." For an interesting interpretation of this title change, see (Pasinetti, 1977).

[5] That is to say to allow multiple primary factors and joints production.

primary factor. The final demand for labor is assumed equal to zero ($C_{n+1} = 0$), and $X_{n+1} = 0 + \sum_{j=1}^{n} x_{n+1\,j}$.

Production of each good depends on a homogenous production function $F_i(.)$ of first degree (constant returns to scale). There is no joint production, thus this is a one-commodity-per-industry economy: $X_i = F_i(x_{1i}, x_{2i}, ..., x_{n+1\,i})$. Since factor substitution is assumed, technical coefficients are not constant, and we deduce from the production function the following simple relation: $F_i\left(a_{1i}, a_{2i}, ..., a_{ni}\right) = 1$.

Given technical constraints and the available quantity of labor, and given the final demand for other industries, equilibrium is determined for each industry *k* in such a way that the final demand for industry *k* (C_k) is maximized. Each industry *k*, is ruled by the following optimization program:

$$\begin{cases} Max\, C_k = X_k - \sum_{j=1}^{n}{}_{k \neq j}\, x_{kj} \\ X_k = F_k(x_{1k}, x_{2k}, ..., x_{n+1\,\kappa}) \\ F_i(x_{1i}, x_{2i}, ..., x_{n+1\,i}) - \sum_{j=1}^{n} x_{ij} = C_i \quad \forall i = 1, ..., n \;\; et \;\; i \neq k \\ X_{n+1} = 0 + \sum_{j=1}^{n} x_{n+1\,j} \end{cases}$$

The solution to the program is found using the Lagrange multipliers. One shows easily that technical coefficients are *independent* of the C_i (final demands) of the prices and of X_{n+1} (the available quantity of labor). As a consequence, even if factor substitution is possible, i.e., technical coefficients are not constant in such specific conditions (one primary factor, no joint production), the technical coefficient will not change whatever change occurs in final demand, prices, and availability of the primary factor. Hence, constancy of technical coefficients is a particular case of the general case where joint production and multiple primary factors allow changes in the structure of production.

According to Samuelson, the non-substitution theorem reconciles the model of Leontief (constant technical coefficients and the value-added distribution is given) with the theory of general equilibrium (variable technical coefficients and the theory of marginal productivity rules income distribution). For Samuelson, “actually all his [Leontief’s] theory in its present form is compatible with the more general case of substitutability" (Samuelson, 1951, p.142–143). In other words, the situation described by the model of Leontief isn't contradictory with and not exclusive of the theory of general equilibrium: it is a particular case of the Walrasian theory of general equilibrium. Indeed, Samuelson's NST means that under certain circumstances "all desirable substitutions have already been made by the competitive market, and no variation in the composition of final output or in the total quantity of labor will give rise to price change or substitution" (*Ibid.* p.143); otherwise, whether the factors of production are substitutable, a change in final demand, relative prices, or in available labor, substitutions between factors occur and technical coefficients change. In a nutshell, the theorem raises the question of the validity of a price-quantity equilibrium independent of consumers' tastes and of the structure of demand.

From an epistemological point of view, the non-substitution theorem is powerful because it makes an initially postulated assumption (the constancy of technical coefficients) the *consequence* of a set of reduced assumptions. Indeed, while Leontief assumed the constancy of technical coefficients, Samuelson showed that in a generalized model of Leontief the constancy is true under precise conditions (only one primary factor, no joint production, etc.). In other words, in the generalized model of Leontief, the constancy is the result of a deduction from fundamental assumptions. From a logical point of view, the status of the theoretical proposition concerning the constancy of technical coefficients changes in the generalized model: this is no more an initial assumption but the result of a deduction (a theorem). We will show now that this theorem is an "operational" theorem according to Samuelson's operationism definition.

2. Deductive Methodology and Operationism: A Cambridge Methodology

From the point of view of theoretical political economy, Samuelson's non-substitution theorem is a strategic piece of the hegemonic ambitions of the general equilibrium theory. Now, we aim at examining the NST's epistemological status.

Samuelson's methodology is well-known. The methodological criterion he chose to change the scientific vocabulary of economics was an operational one he defined in 1938 (Samuelson 1938a). However, when using the operational criterion, Samuelson wasn't alone in doing so. The term 'operational' was common in Cambridge and especially in Harvard. Indeed, while empirical and experimental studies had been developing in social sciences since the end of the 19th century, there were very different ways to articulate theory and observations, even under the influence of the Vienna circle: the operational methodology is one of them.

In economics, Henry Schultz (1928) was the first to allude to an *operational* methodology. According to Schultz, his demand-estimation method was "an illustration of what…Bridgman…has called 'the operational procedure' for determining the meaning of a concept [in this case the meaning of demand]" (1928, p.647). Indeed, the physicist Percy W. Bridgman, from Harvard University, developed in his book *Logic of Modern Physics* (1927) the philosophical doctrine he called *operationalism* (also called *operationism* or *operational analysis*; see Bridgman, 1938). This doctrine states that the meaning of a scientific concept is synonymous with the set of operations entering its definition.[6] In other words, the meaning of a concept is given by an explicit definition of this concept stating experimental or mental operations of measure. This is the experimental version (and not the *mentalist* one) of Bridgman's operationalism that was successfully exported in social sciences, psychology, and philosophy.[7]

One particularity of operationism is that it proposes a strong relationship (synonymy) between theoretical terms and observational terms (direct observations or sense data), without distinction between induction and deduction. Nevertheless, in contrast to induction, operationism considers theory as necessary *and* requires theoretical terms to be operational, that is to say to be empirically meaningful (while induction is suspicious about theory). In the philosophy of science, Rudolf Carnap (1936, 1937),[8] showed that operationism was actually the strong version of neopositivist verificationism. Carnap stated at the same time that operationism was impracticable. For the philosophy of science, Carnap's article ended the operational program, although Carl Gustav Hempel, Karl Popper, and Nelson Goodman continued to criticize and to discuss operationism during the forties and the fifties. It also put an end to the until-then-flourishing verificationist program. This led to the creation of what Carnap called *logical empiricism.*[9] Despite the doom of operationism in pure philosophy of science, the very spirit of Bridgman's methodology became very fashionable in American sciences. In Cambridge (MA) again, first Samuelson (in 1938a), a student of Leontief at

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[6] In Bridgman's words: "We mean by any concept nothing more than a set of operations; the concept is synonymous with the corresponding set of operations" (Bridgman 1927, 5, our emphasis). In economics, Schultz noticed: "As long as our fundamental concepts are not synonymous with corresponding sets of operations (and, hence, do not admit of the possibility of experimental verification), it is wrong to speak of economics as an experimental science." (Schultz, 1928, 647)

[7] Behaviourism is particularly influenced by operationism.

[8] Carnap, a German citizen, emigrated in 1935 from Czechoslovakia to the United-States. Though Carnap was a professor of philosophy at the University of Chicago (1936-1952), he spent the years 1939-1941 at Harvard as a visiting professor. From 1934 on, Carnap has been close to the Harvard-professor Willard Van Orman Quine.

Harvard and then working at the MIT, and later Leontief (in 1949), at Harvard, alluded to the operational criterion.

3. Samuelson's Operational Theorems

Samuelson first studied the neoclassical theory of demand and utility and raised the question: if one denies to the "hedonistic, introspective, psychological elements" explanatory value, "does not the whole utility analysis become meaningless in the *operational* sense of modern science?" (Samuelson, 1938a, p.344, our emphasis). The *Foundations of Economic Analysis* (1947) aimed at being a translation of economic theory into the operational language of science. For Samuelson, "A meaningless theory according to this criterion is one which has no empirical implications by which it could be *refuted* under ideal empirical conditions" (*Ibid*., our emphasis); and again, "It is the purpose here to demonstrate that the utility analysis in its ordinary form does contain empirically meaningful implications by which it could be *refuted*." (1938a, 345, our emphasis)

The definition Samuelson gave of his operational criteria was stable in time and he didn't remove it. In 1998, Samuelson stated the following:

> My approach looked backward in summarizing "economically" (in the Mach-Vienna Circle sense) the "meaningful" (testable and, in principle, refutable) core of constrained-budget demand theory (Samuelson, 1998, p.1380).

Samuelson's project was to eliminate from "the theory of consumer behavior any 'vestigial traces of the utility concept,' consequently stripping consumption theory 'to its bare implications for empirical realism'" (Samuelson, 1938b, p.61). According to Samuelson, this is that method that permits to enounce *operational theorems*.

Now, it is important to underline that the definition Samuelson gave of the operational meaning of a theory is different from both Bridgman's definition and Carnap's version of operationism. Compared to Carnap and Bridgman's approaches, it is controversial to state whether the methodology Samuelson adopted was based on a *meaning* criterion or a *falsification* criterion (see Blaug, 1980; Cohen, 1995; Mongin, 2000; Akhabbar, 2007a, 2007b; Hands, 2008). Blaug (1980) argued that Samuelson's operationalism was a mixture of empiricism and falsificationism.[10] Although commentators agree about the radical difference

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[9] We consider in what follows that Carnap's version of operationism is the general one (and not Bridgman's).

[10] Karl Popper is considered as the father of falsificationism. This methodology doesn't permit to find the empirical meaning of a concept but only to say if it is a scientific one and if it is not yet falsified.

between Bridgman's and Samuelson's operationism, there is no consensus about the falsificationist nature of the latter: for instance, Cohen (1995) rejected Blaug's position. To reject the falsificationist interpretation of Samuelson's operationist methodology, Cohen argues that Popper's book *Die Logik der Firschung* (The Logic of Scientific Discovery), published in 1934, was translated in English late in the fifties (1959), and he assumes that Samuelson didn't read it. Now, on a pure epistemological stage, Samuelson's definition is clearly not the one of operational methodology, but is a version of falsificationism: according to such a methodology, theory doesn't need direct correspondence with data but only indirect empirical implications in order to be (possibly) refuted.

To our demonstration about the operationality of the NST, it is not important to determine or choose one or another interpretation of Samuelson's epistemology. This will not affect our result. From now, we will consider that operationism is in the first place the methodology Samuelson practically implemented in revealed preference theory (RPT), and we will show that the same methodology was implemented in the NST.

4. Operational Theorems: From Revealed Preference Theory to NST

Samuelson first applied his so-called operationist methodology to (neoclassical) consumer theory. To bring empirical implications to the theory of consumer, Samuelson chose to start his analyses with the data consumers reveal in the marketplace. The utility function is then the one maximized that corresponds to the revealed preference. This function is a transitive and complete ordinal utility function, which satisfies the weak axiom of revealed preference. Ordinal utility is then purged from introspection and gets an operational meaning, according to Samuelson: the function is defined "as that which is maximized by means of *revealing preferences* in the market place" (Cohen, 1995, p.66).

This is clearly the same principle employed in the theory of revealed preference and the non-substitution theorem: given prices—assumed to be "parameters not subject to influence by the individual" noticed Samuelson (1938b, 62)—the explanation of empirical data, namely actual technical coefficients (or *consumed quantities*) is expressed as an optimization program[11] of a derivable mathematical function, namely final net output (or *utility function*) under constraints, here the production function and the quantities equilibrium (or *budgetary constraint*). Let's note that in the NST, the production function actually appears in the objective function since, for each industry *k*, one maximizes consumption C_k where

$C_k = (F_k(X) - \sum x_{kj})$, *F(-)* being the production function. Therefore, in the NST, like in revealed preference theory, the unknown objective function is to be found. In both optimization problems, to solve the program, the method of Lagrangean multipliers is used.

On the one hand, the non-substitution theorem shows that a production function with substitution exists that induces under certain circumstances constant technical coefficients (one *observes* constant technical coefficients) and that induces under other circumstances variable technical coefficients (one *observes* the variation of technical coefficients). On the other hand, in the same way as the production function obtains an operational meaning by its references to variations of measurable technical coefficients, the utility function gains empirical significance once leaving mysterious allusions to introspection.[12]

In microeconomics textbook language, Samuelson's analysis starts with observable point, namely bundles of goods. In the entirety of this paragraph, we assume given prices. In the realm of production theory, the problem is as follows: given observable bundles of inputs, what is the corresponding operational production function? In the realm of consumer theory, the problem is as follows: given observable bundles of goods, what is the corresponding operational utility function? Indeed, given prices, on a two-axis diagram one represents quantities of input *A* and of input *B* required to produce *x* units of commodity *X*; this point gives a measure of one technical coefficient of the production function of commodity *X*. One may represent the isoquant passing by this observable point. The isoquant is not observable in reality while the technical coefficient may be measured. Given assumptions on the production function and given a series of observable input vectors (*A*,*B*), one deduces the (derivable) production function, $x=F(A,B)$. This is the operational analysis of the production function. In the same way, when one deals with consumer acts, given prices, one observes and measures the consumed quantities of commodities *X* and *Y*. On a two-axis diagram, one represents consumed quantities of *X* and consumed quantities of *Y*. One may represent the indifference curve passing by this observable point (*X*,*Y*). The indifference curve is not observable in reality while the consumed bundle of goods *X* and *Y* is observable. Given assumptions on the utility function and given a series of observable bundles of goods (*X*,*Y*), one deduces the (derivable) utility function, $u=G(X,Y)$.

[11] About the epistemological status of the optimization process, see Samuelson (1972).
[12] Samuelson's methodology is understandable as a methodology concerning the validity of assumptions. Here, he replaced an unoperational assumption (a function of utility based on introspection) by an operational assumption (a utility function based on market data).

These functions (the production function and the utility function) are operational by means of indirect references to observation: this is the deductive method and not the canonical operational method of Carnap and Bridgman. Indeed, Carnap's operational methodology requires direct correspondences between concepts and observation: to gain an operational meaning the mathematical function must be defined by synonymous measure operations… which is not the case.[13] But the mathematical functions obtained by Samuelson are operational according to Samuelson's operational criterion. Samuelson's methodology lies on two stages: (1) enunciation of minimal ('economical') assumptions under which rest deduction and calculation and (2) expression of the explanation of the problem as an optimization program. Ultimately then, the non-substitution theorem is operational according to Samuelson's definition of the operational meaning of a theory, but this definition is completely different from the one Carnap proposed. On the other hand, Samuelson's criterion is falsificationist (close to Popper) and corresponds to the one proposed by Koopmans (1947) at the *Cowles Commission*: the empirical content (and not operational meaning) of a hypothetic-deductive theory comes from the singular statements (observable) deduced from the general assumptions.

5. Conclusion: Samuelson and a Symmetric Operationist Theory of Value? Operational Theorems for Consumption Theory and Production Theory

In this article we have shown that Samuelson applied the same methodological approach to both the economic theory of consumption and the economic theory of production: he intended to offer operational concepts to build economic theory on new and operational foundations. On the consumer theory side, Samuelson produced the revealed preference theory as the operational theory of ordinalist utility analysis. As such, he aimed at getting rid of the psychological basis of utility theory: the utility function was nothing more than an artifact reproducing the observable consumed bundles of goods. On the production theory side, Samuelson grounded controversial-factor substitutability on (expectedly) uncontroversial operational-production functions: the opposition between, on the one hand, classical and Marxian political economy (rejecting substitutability and assuming complementary factors), and so-called neoclassical theories was expected to vanish under the effect of operational theorems of production, beginning with the non-substitution theorem. True or false, the non-substitution theorem is operational.

[13] As noticed above, in contrast to Bridgman's operationism, Samuelson's methodology is not based on a meaning criterion.

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